Length-Weight Relationships for Six Species of Billfishes in the Central Pacific Ocean

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ABSTRACT

Weight-length relationships for six species of billfishes in the central Pacific Ocean were developed by analyzing 20 yr of data. Log-linear and nonlinear statistical models were fitted to the data by regression analysis, and residuals from the models were tested. Blue marlin, Makaira nigricans Lacépède, (50-135 cm FL), male blue marlin (≥135 cm FL) and sailfish, Istiophorus platypterus (Shaw and Nodder), apparently have coefficients of allometry less than 3.0. Black marlin, M. indica (Cuvier) and female blue marlin (≥135 cm FL) apparently have coefficients equal to 3.0. Shortbill spearfish, Tetrapturus angustirostris Tanaka, striped marlin, T. audax (Philippi), and swordfish, Xiphias gladius Linnaeus, apparently have coefficients greater than 3.0.

As with most studies on the length-weight relationship, this study is not an end in itself. It was initiated to provide length-weight conversion relationships (Equation 1) for use in a growth paper on blue and striped marlins (Skillman and Yong2), as well as to provide conversion charts for the sport fishermen at the Hawaiian International Billfish Tournament. There are few published papers on the weight-length relationship of billfishes3 (de Sylva, 1957; Royce, 1957; Kume and Joseph, 1969); hence, we decided to calculate this relationship for all six species of billfishes on which data had been collected by the Honolulu Laboratory of the Southwest Fisheries Center, National Marine Fisheries Service. These six species were the black marlin, Makaira indica (Cuvier), blue marlin, M. nigricans Lacépède, sailfish, Istiophorus platypterus (Shaw and Nodder), shortbill spearfish, Tetrapturus angustirostris Tanaka, striped marlin, T. audax (Philippi), and swordfish, Xiphias gladius Linnaeus.

Although all of the length-weight data collected on billfishes from 1950 to 1971 by the Honolulu Laboratory were used, this study should not be considered exhaustive or definitive. Even in the best represented species, there were too few data to separate the data according to sex, maturity, and season as suggested by Le Cren (1951) and Tesch (1968). Thus, it was impossible to perform a detailed analysis of covariance similar to that performed recently by Brown and Hennemuth (1971) on haddock, Melanogrammus aeglefinus (Linnaeus). Some species were so poorly represented that the length-weight relationships should be considered as tentative relationships.

In general, fishery biologists have accepted the appropriateness of the allometric growth equation (Huxley and Teissier, 1936) or its mathematical equivalent, the power function, as a descriptor of growth in weight to growth in length. We accepted the general form of the relationship (Equation 1) and applied both the log-linear and the nonlinear statistical

\[ W_i = b_1 L_i^{a_i} \] (1)

models of the relationship. Each model is discussed, and statistical procedures for evaluating the goodness of fit are presented. Papers by Glass (1969), Pienaar and Thomson (1969), and Hafley (1969) are particularly relevant to this discussion.

MATERIALS AND METHODS

Collection of Data

The data used in this report came from three sources. In nearly all of them fork length (FL) measurements were taken to the nearest centimeter.
from the tip of the snout to the fork of the tail. Where naris or eye-orbit fork length measures were given, conversion to FL was performed with equations given by Royce (1957). All weight measurements were taken to the nearest pound and were converted to kilograms before analysis.

Two of the data sets were derived from longline catch records taken by research vessels of the Honolulu Laboratory while fishing in central Pacific waters, mostly near the equator. The first of these data sets (deck 1) was obtained from a morphometric study of billfishes by Royce (1957) that was carried out on a series of longline cruises in 1950 to 1953. The second data set (deck 2) was obtained from routine information collected from longline-caught fishes for the years 1950 to 1971. These two longline data sets were combined in the subsequent analyses because they represent the same type of data, though they were collected for different reasons and, in general, do not overlap in time. The last set of data (deck 3) was collected by personnel of the Honolulu Laboratory from fish caught by trolling between 1962 and 1971. in June (once), July, or August during the Hawaiian International Billfish Tournament held in Kailua-Kona, Hawaii (Table 1). Since the five species other than blue marlin were represented in such small numbers in the sample, they were pooled with the longline data. For blue marlin, the trolling-derived data were analyzed separately from the longline-derived data. The longline data represent a pooling over all seasons of oceanic-caught fish while the trolling data represent only inshore catches during the summer months.

All three data sets for most species contained some determinations of sex and maturity, but only the trolling data (deck 3) for blue marlin contained enough information to allow an examination of the sexes separately. All other species and pooled data sets were examined without regard to the sex of the individuals.

Analysis

The goal of this paper was to obtain length-weight relationships for each species by using a statistical model that fitted the data best. To accomplish this goal, the steps listed below were followed:

1. The data were checked for different growth stanzas by plotting the natural logarithms of weight against the natural logarithms of length.
2. Length-weight relationships using log-linear regression for weight on length were obtained for all species.
3. The normality of the error terms was tested for those species that had enough data to perform the tests.
4. The log-linear relationships were tested for their significance.
5. Length-weight relationships using nonlinear regression of weight on length were obtained for blue and striped marlins.
6. Statistical tests were performed to determine whether the log-linear or the nonlinear model was more appropriate.
7. The coefficients of allometry were tested to see if they were different from 3.0.

In subsequent paragraphs, brief discussions will be given regarding adjustments made for the amount of data available for each species, the statistical models themselves, the criteria used to determine best fit, and certain test statistics employed in the analysis.

As can be seen from Table 1, the amount of data available for most of the species for any data deck was very small. Even after pooling all of the data for the black marlin, sailfish, shortbill spearfish, and swordfish, there were too few data to evaluate the fit of the statistical models. Hence, the most commonly used statistical model, the log-linear, was fitted to these species. Only the significance of the relationships was tested. For striped marlin after pooling all data, there were enough data to evaluate the fit of the statistical models. In the analysis of blue marlin, the data were not pooled because we believed that the longline- and troll-derived data represented different biological situations. The longline data were obtained from a sampling program that neglected any seasonally varying and sexually different length-weight relationships, whereas the troll data were obtained in the summer season for each sex. To aid in the interpretation of the striped marlin data, the blue marlin data were pooled for comparative purposes only. There were enough data to evaluate the fit of the models for all blue marlin data categories.

As mentioned in the introduction of this paper, fishery biologists, in general, have accepted the appropriateness of the allometric growth equation as a descriptor of the growth in weight to the growth in length of fish. As expressed by Equation 1, this equation is mathematically a functional relationship (Madansky, 1959) where weight is known exactly
Table 1.—Number of observations by species, by year, by data deck. Where deck 1 is from Royce (1957), deck 2 is the Honolulu Laboratory's longline punch card deck, and deck 3 is from the Hawaiian International Billfish Tournament.

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from a given length; this is not a biologically reasonable model. Traditionally, length has been viewed as the independent variable that is measured with no error and weight as the random dependent variable that is measured with error. The validity of these assumptions is beyond the scope of this paper and will not be discussed. We have concerned ourselves with the appropriateness of two statistical models, the log-linear and nonlinear models. The log-linear model, with log-additive error, was written as

\[ \ln W_i = \ln h_i + a_i \ln L_i + \ln \varepsilon_{\ln}, \]

(2)

The arithmetic equivalent of this model can be written as

\[ W_i = h_i L_i^{a_i} \varepsilon_{\ln}, \]

but this equation should not be construed to be the model. The nonlinear model, with additive error, was written as

\[ W_i = h_i L_i^{a_i} + \varepsilon_{\ln}, \]

(3)

The evaluation of the goodness of fit of regression lines can be divided into distinct tests of precision (or significance) of the regression and of the appropriateness of the model. The appropriateness of a model (Equation 2 or 3) was tentatively accepted, and the model was fitted to the data. The precision of this fit can then be measured by the "F" test and the "f" test, both of which test \( H_N: \varepsilon_i = 0 \) and \( H_A: \varepsilon_i \neq 0 \), or the "R~" statistic, the "proportion of total variation about the mean explained by regression" (Draper and Smith, 1966). All of these tests are equivalent and basically measure the usefulness of the regression as a predictor.

To be able to perform any of these tests, the random error term must be normally distributed. The distribution of \( \varepsilon_i = \ln \varepsilon_{\ln} \) was tested for the log-linear model by calculating R.A. Fisher's statistics for skewness (G1) and kurtosis (G2, measuring the amount of peakness or bimodality). A model can fail in the significance tests because the model is incorrect or because the sample size is small relative to the amount of variability in the data. In addition, if a model is nonlinear in its parameters, it is not possible to test for significance because the variance estimates are biased, making it superfluous to test the distribution of the error term, \( \varepsilon_{\ln} \).

Moreover, the residual sums of squares for linear and nonlinear least squares fitting routines cannot be compared because they are minimal estimates in their respective sample spaces. We chose to present the "R~" and "F" statistics for the log-linear model as an indication of precision, but did not use the statistics in deciding best fit, since they cannot be compared to those obtained for the nonlinear model.

Our criteria for best fit of the models were based on measures of appropriateness, namely, whether the error terms have the following properties:

\[ E[\varepsilon_{\ln}] = 0 \text{ or } E[\varepsilon_{\ln}] = 0 \]
\[ \text{Var}(\varepsilon_{\ln}) = \sigma_{\ln}^2 \text{ or } \text{Var}(\varepsilon_{\ln}) = \sigma_{\ln}^2. \]

(4)

that is, the error terms have a mean equal to zero and a constant variance. The error terms for the log-linear model must have a mean equal to zero, since an intercept term was included in the model (Draper and Smith, 1966, p. 87). For the nonlinear model, it is not readily apparent that the error term must be equal to zero; hence, the mean was calculated. The residuals were plotted against the dependent and independent variables to check for constant variance. If variance is constant, the residuals appear as a horizontal band along the variable axes (Draper and Smith, 1966, p. 86).

The final regression coefficients, or coefficients of allometry, were tested using the hypothesis scheme \( H_N: a = 3.0 \) and \( H_A: a \neq 3.0 \) (Steel and Torrie, 1960, p. 171).

In reporting the results of the various statistical tests, the following convention was used: "NS" indicates not significant at the 0.05 level, "*, **, ***" indicate significance at the 0.05, 0.01, 0.001 levels, respectively; and "d.f." stands for degrees of freedom.

RESULTS

Growth Stanzas

The weight-length data for each species were first plotted with logarithms of weight versus logarithms of fork length in order to subjectively check for more than one growth stanza (Tesch, 1968). Blue marlin

\[ \text{From this statement, the estimated value of the log-error term, } \varepsilon_{\ln}, \text{ may be taken as zero which in turn indicates that } \varepsilon_{\ln} \text{ in the arithmetic equivalent to the log-linear model (Equation 2) may be taken as equal to one. If the arithmetic equivalent to the log-linear model were designated as a separate model, it does not follow that } E[\varepsilon_{\ln}] = 1 \text{ or that } \text{Var}(\varepsilon_{\ln}) = \sigma_{\ln}^2. \]
Table 2.—Weight-length relationships for billfishes using the log-linear model (Equation 2). The pooled category under the data set heading indicates pooling of longline and trolling data. Dashes indicate that statistical tests were not appropriate. The pooled data for blue marlin includes trolling-derived data for which sex was not determined.

<table>
<thead>
<tr>
<th>Species</th>
<th>Data set</th>
<th>Sample size (N)</th>
<th>$b$</th>
<th>$a$</th>
<th>Variance</th>
<th>$R^2$ in percent</th>
<th>$\ln W$, $\ln L$</th>
<th>$a$</th>
<th>$b$</th>
<th>$G^1$</th>
<th>$G^2$</th>
<th>$F^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black marlin</td>
<td>Pooled</td>
<td>24</td>
<td>2.3787 x 10^{-4}</td>
<td>3.1654</td>
<td>97.2</td>
<td>0.0069</td>
<td>0.0131</td>
<td>0.4134</td>
<td>—</td>
<td>—</td>
<td>766.36***</td>
<td></td>
</tr>
<tr>
<td>Blue marlin</td>
<td>Longline</td>
<td>4</td>
<td>5.1827 x 10^{-1}</td>
<td>0.6678</td>
<td>96.4</td>
<td>0.0072</td>
<td>0.0084</td>
<td>0.1654</td>
<td>—</td>
<td>—</td>
<td>52.93*</td>
<td></td>
</tr>
<tr>
<td>Blue marlin 50-135 cm FL</td>
<td>Pooled</td>
<td>545</td>
<td>5.0048 x 10^{-4}</td>
<td>3.0214</td>
<td>95.0</td>
<td>0.0126</td>
<td>0.0326</td>
<td>0.0011</td>
<td>-0.29**</td>
<td>1.77**</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>Blue marlin &gt;135 cm FL</td>
<td>Pooled</td>
<td>148</td>
<td>4.7226 x 10^{-6}</td>
<td>3.0442</td>
<td>96.2</td>
<td>0.0172</td>
<td>0.0055</td>
<td>0.1672</td>
<td>0.13 NS</td>
<td>—</td>
<td>1.677.27***</td>
<td></td>
</tr>
<tr>
<td>Trolling</td>
<td>338</td>
<td>4.2906 x 10^{-4}</td>
<td>3.0470</td>
<td>94.9</td>
<td>0.0111</td>
<td>0.0013</td>
<td>0.0406</td>
<td>-0.58**</td>
<td>2.09**</td>
<td>—</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>Trolling (male)</td>
<td>276</td>
<td>2.2929 x 10^{-5}</td>
<td>2.7405</td>
<td>82.7</td>
<td>0.0118</td>
<td>0.0057</td>
<td>0.1719</td>
<td>-0.72**</td>
<td>2.62*</td>
<td>—</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>Trolling (female)</td>
<td>187</td>
<td>3.9820 x 10^{-6}</td>
<td>3.0611</td>
<td>92.0</td>
<td>0.0153</td>
<td>0.0096</td>
<td>0.3171</td>
<td>-0.73**</td>
<td>1.95*</td>
<td>—</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>Trolling (female)</td>
<td>85</td>
<td>1.9445 x 10^{-8}</td>
<td>3.1871</td>
<td>93.2</td>
<td>0.0127</td>
<td>0.0089</td>
<td>0.2914</td>
<td>-0.12 NS</td>
<td>-0.29 NS</td>
<td>1.144.57***</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>Shortbill spearfish</td>
<td>Pooled</td>
<td>16</td>
<td>5.0083 x 10^{-5}</td>
<td>3.8338</td>
<td>65.2</td>
<td>0.0332</td>
<td>0.5596</td>
<td>14.3795</td>
<td>—</td>
<td>—</td>
<td>26.27***</td>
<td></td>
</tr>
<tr>
<td>Striped marlin</td>
<td>Pooled</td>
<td>53</td>
<td>5.7126 x 10^{-7}</td>
<td>3.3756</td>
<td>93.1</td>
<td>0.0336</td>
<td>0.0166</td>
<td>0.4723</td>
<td>1.14**</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Swordfish</td>
<td>Pooled</td>
<td>7</td>
<td>2.3296 x 10^{-7}</td>
<td>3.5305</td>
<td>98.9</td>
<td>0.0169</td>
<td>0.0286</td>
<td>0.8401</td>
<td>—</td>
<td>—</td>
<td>435.34***</td>
<td></td>
</tr>
</tbody>
</table>

*Indicates significance at the 0.001 level. ** Indicates significance at the 0.01 level. NS indicates not significant at the 0.05 level. * Indicates significance at the 0.05 level.  
These data sets include the same single aberrant datum.
was the only species exhibiting such a trend (Fig. 1) and then only for the longline-caught fish. Although it was quite evident that two growth stanzas existed, there were too few data to determine exactly where the two stanzas met or overlapped. We arbitrarily took the two data points at 135 cm FL (4.9 in natural logarithms) as the overlap area, with the assumption that the length-weight relationship for the older, well-represented stanza should be accurately predicted even if it actually began at a smaller size while that for the younger stanza is provisional. The younger growth stanza was treated separately in the subsequent analyses.

Table 3.—Weight-length relationships for blue and striped marlins using the nonlinear model (Equation 3). The data sets pooled category indicates pooling of longline and trolling data.

<table>
<thead>
<tr>
<th>Species</th>
<th>Data set</th>
<th>Sample size (N)</th>
<th>$b$</th>
<th>$a$</th>
<th>$R^2$ in percent</th>
<th>$\hat{c}$</th>
<th>G1</th>
<th>G2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue marlin</td>
<td>Pooled</td>
<td>453</td>
<td>6.3087 x 10^-1</td>
<td>2.9827</td>
<td>93.1</td>
<td>-0.5717</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Longline</td>
<td>68</td>
<td>3.9290 x 10^-4</td>
<td>3.0821</td>
<td>94.4</td>
<td>1.1889</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Trolling</td>
<td>385</td>
<td>8.5300 x 10^-6</td>
<td>2.9265</td>
<td>92.2</td>
<td>0.6549</td>
<td>2.999*</td>
<td>36.691**</td>
</tr>
<tr>
<td></td>
<td>Trolling</td>
<td>276</td>
<td>1.9421 x 10^-4</td>
<td>3.1895</td>
<td>98.9</td>
<td>0.3003</td>
<td>0.266*</td>
<td>3.723**</td>
</tr>
<tr>
<td></td>
<td>Trolling (male)</td>
<td>276</td>
<td>18.9972 x 10^-4</td>
<td>2.7756</td>
<td>83.1</td>
<td>0.1438</td>
<td>0.121 NS</td>
<td>2.894**</td>
</tr>
<tr>
<td></td>
<td>Trolling (female)</td>
<td>86</td>
<td>4.8246 x 10^-4</td>
<td>3.0249</td>
<td>90.8</td>
<td>0.4055</td>
<td>2.991*</td>
<td>20.499**</td>
</tr>
<tr>
<td></td>
<td>Trolling</td>
<td>85</td>
<td>1.7082 x 10^-4</td>
<td>3.2111</td>
<td>91.9</td>
<td>0.1341</td>
<td>-0.067 NS</td>
<td>0.577 NS</td>
</tr>
<tr>
<td>Striped marlin</td>
<td>Pooled</td>
<td>53</td>
<td>1.0973 x 10^-1</td>
<td>3.2589</td>
<td>90.7</td>
<td>0.1553</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

** indicates significance at the 0.01 level, * indicates significance at the 0.05 level, and NS indicates not significant at the 0.05 level.
however, the elimination of this datum did not alter the results significantly. When the trolling data were divided into males and females, the error terms were still not normally distributed. However, when the above mentioned aberrant datum for the female data was dropped from the calculations, the error terms were normally distributed. The "F" test showed that the relationship was highly significant, and the relationship accounted for 93% of the variation in the data.

For large blue marlin (five relationships) and striped marlin, the residuals about the regression line were plotted against the dependent (weight) and independent (fork length) variables in order to evaluate the fit of the log-linear model. In every case, the distribution of the residuals appeared as a band along the axes; hence, the model appeared to fit the data. The results for striped marlin and blue marlin (trolling data for females with all data points and with the one aberrant datum point dropped) were representative of all the species plots. These results are presented in Figure 2. The two plots for the blue marlin indicated the effect of the aberrant datum that was discussed earlier when the normality of the residuals was tested. In spite of the residuals not being normally distributed for all except two of
the cases (Table 2), the plotting of the residuals indicated that there was no reason to reject the assumption of constant variance. Hence, the log-linear model seemed to be appropriate.

**Nonlinear Model**

The nonlinear model (Equation 3) was fitted to the data for the large blue marlin (five relationships) and the striped marlin (Table 3) in order to compare the fit of this model to that for the log-linear model. Since the estimate of \( \sigma^2 \) is biased in nonlinear regression and therefore tests of significance cannot be made, the distribution of the error terms was not tested. The estimates of \( \ell^{\hat{R}^2} \) (a biased estimator in this nonlinear case) indicated that the nonlinear model does not in general account for as much of the variation in the data and is, therefore, not as good a predictor as the log-linear model. When the residuals from the nonlinear regression lines were plotted against the dependent and independent variables, it was found in every case that the amount of error was small for small values of the variables and large for large values of the variables. Hence, the assumption

![Figure 3](image-url)

*Figure 3.*—Plot of residuals from the nonlinear model for female blue marlin with 86 and 85 samples and for striped marlin with 53 samples.
of constant variance of the error term must be rejected for all cases. The results for blue marlin, trolling data for females with 86 and 85 data points, and for striped marlin presented in Figure 3 were representative of all species plots. Comparing these plots with those in Figure 2 showed that the nonlinear model did not fit the data as well as did the log-linear model. Since both assumptions regarding the properties of the error terms were rejected, it must be concluded that the nonlinear model is not appropriate for these sets of data.

**Coefficients of Allometry**

The coefficients of allometry that will be discussed in this section were obtained from the fitting of the log-linear model. For those species and data sets in Table 2 where the assumption of normality of the residuals was rejected, the coefficients of allometry were not tested. The hypotheses tested were $H_N: a = 3.0$ and $H_A: a \neq 3.0$ (a two-sided "t" test), and the results of these tests are presented in Table 4. For small blue marlin and swordfish, the null hypothesis that $a = 3.0$ was rejected on the basis of the data available. For black marlin, large blue marlin (longline data), female blue marlin, sailfish, and shortbill spearfish, the alternate hypothesis that $a \neq 3.0$ was rejected on the basis of the data available.

**DISCUSSION**

Weight-length relationships were fitted successfully for all six species of billfishes appearing in the Honolulu Laboratory's collections (Figs. 4 and 5). The log-linear relationships (Table 2) were found to be more appropriate than the nonlinear relationships (Table 3) for every species and data set. The significance of all the relationships was not testable since many of the error terms were not normally distributed; however, the "$R^2$" values indicated that all of the relationships, except for the shortbill spearfish, account for a high percentage of the variance in the data. Hence, on the basis of fit and amount of variance accounted for, these relationships should be good predictors.

However, the usefulness of the relationships as predictors also varies according to the amount of

![Figure 4](image-url).—Weight-length relationships using the log-linear model for swordfish, shortbill spearfish, sailfish, black marlin, and striped marlin.
Table 4.—Final weight-length relationships using the log-linear model, $W = \beta L^\alpha$, for the indicated data sets. $H_N$ indicates the null hypothesis tested, and the accompanying alternate hypothesis was then $H_A: \alpha \neq 3.0$. The data for the eastern tropical Pacific were obtained from Kume and Joseph (1969). Dashes indicate that the test could not validly be performed. Size ranges are in centimeters fork length.

<table>
<thead>
<tr>
<th>Species</th>
<th>Data set</th>
<th>Sample size (N)</th>
<th>$b$</th>
<th>$a$</th>
<th>$t$ for $H_N: \alpha = 3.0$</th>
<th>Size range</th>
<th>Eastern Tropical Pacific</th>
<th>$b$</th>
<th>$a$</th>
<th>Size range</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Black marlin</td>
<td>Pooled</td>
<td>24</td>
<td>$2.3787 \times 10^{-6}$</td>
<td>3.1654</td>
<td>1.447 NS</td>
<td>214.5-373.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Blue marlin</td>
<td>Longline</td>
<td>4</td>
<td>$5.1827 \times 10^{-1}$</td>
<td>0.6678</td>
<td>$-25.410^*$</td>
<td>50.0-135.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Blue marlin</td>
<td>Pooled</td>
<td>453</td>
<td>$5.0048 \times 10^{-6}$</td>
<td>3.0214</td>
<td>NS</td>
<td>135.0-456.9</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sailfish</td>
<td>Pooled</td>
<td>18</td>
<td>$2.0739 \times 10^{-3}$</td>
<td>2.6504</td>
<td>$1.429^*$</td>
<td>177.0-281.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shortbill spearfish</td>
<td>Pooled</td>
<td>16</td>
<td>$5.0083 \times 10^{-6}$</td>
<td>3.8338</td>
<td>$1.115^*$</td>
<td>140.0-180.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Striped marlin</td>
<td>Pooled</td>
<td>53</td>
<td>$5.7126 \times 10^{-7}$</td>
<td>3.3756</td>
<td>NS</td>
<td>142.2-310.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Swordfish</td>
<td>Pooled</td>
<td>7</td>
<td>$2.3296 \times 10^{-7}$</td>
<td>3.5305</td>
<td>$2.880^*$</td>
<td>145.2-324.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

** indicates significance at the 0.01 level, * indicates significance at the 0.05 level, and NS indicates not significant at the 0.05 level.
data used in the analysis, the range of the data, and whether sexes were analyzed separately. Considering the sample size (4) and the method of selecting the points of overlap, the relationship for small blue marlin (50-135 cm FL) was provisional. The relationship for shortbill spearfish was also provisional since there were 16 data points ranging from 140.0 to 180.0 cm FL. Although the sample sizes for black marlin, sailfish, and swordfish were small (24, 18, and 7, respectively), the ranges were wide, and the relationships should be taken as valid estimates. For striped marlin and for blue marlin, considering all data sets, there were enough data to obtain valid relationships. The importance of the results for the various blue marlin data sets will be discussed in connection with the coefficients of allometry.

Concrete interpretations of the coefficients of allometry are precluded by a statistical inability to test the significance of all the coefficients as well as to test between coefficients of different species or data sets. The coefficient for swordfish was the only one tested that was apparently greater than 3.0. For the other species tested, black marlin, blue marlin (longline data), female blue marlin (trolling data), sailfish, and shortbill spearfish, the hypothesis that the coefficient was equal to 3.0 could not be rejected. That is, the growth in weight to length was isometric for these species. Intuitively, we doubt these results for sailfish and shortbill spearfish and suspect that additional data would show the coefficient for sailfish to be less than isometry and for shortbill spearfish to be greater than isometry.

For blue marlin, the interpretation of the results was complicated by an inability to perform statistical tests of hypotheses. The coefficient of allometry for the small blue marlin indicated that the small fish maintain a very different weight to length growth relationship than do the larger, adult fish. Part of this difference may have been due to differential growth of the bill in the younger fish. It was apparent from Table 4 that there was not a real difference between longline- and troll-caught blue marlin; the coefficients of allometry as well as the intercept "b" were extremely similar. This does not necessarily imply that there are no seasonal differences in the weight-length relationship of blue marlin but does indicate that no such effect could be shown with 68 data points from longline catches made over all seasons. When the trolling data were divided according to sex, it was found that the coefficient for females did not differ significantly from isometry while that for males was probably less than isometry. The male and female curves (Fig. 5) could not be distinguished where the data overlapped. Hence, the increased weight to length growth shown by the females occurs primarily at lengths greater than those attained by males in this

![Figure 5](image-url)
data set. The sexual dimorphism in length that has been noted by many workers (e.g., Strasburg, 1970) apparently extends to the weight-length relationship also. That is, females not only grow to a greater length than males, but are proportionally heavier at the same length.

For striped marlin, analysis of the pooled data produced an estimate of the coefficient of allometry that appears to be greater than isometry. Inability to divide the data by sex was unfortunate since it is not known whether sexually dimorphic growth characteristics exist for the striped marlin. If such an effect does exist, it is believed to be less marked than in the blue marlin. Hence, the largeness of the striped marlin coefficient relative to that for the blue marlin, for both pooled and female data alone, probably was not due to sexual dimorphism.

There are only two papers in the literature giving weight-length relationships that may be compared to ours, since the data used by Royce (1957) were included in this analysis. De Sylva (1957) presented a length-weight plot for sailfish from the Atlantic Ocean, but a model was not fitted to the data. A fish approximately 250 cm FL would weigh 34 kg whereas our study predicts 37 kg. Kume and Joseph (1969) fitted the log-linear model to blue marlin, sailfish, shortbill spearfish, striped marlin, and swordfish data. The coefficients of allometry and the intercept points from their calculations are presented in Table 4 for direct comparison to those from this study. For all species, the coefficients of allometry for fish from the central Pacific were greater than those from the eastern tropical Pacific. If the coefficients were shown to be statistically different, there would be little point in comparing the intercept values since the relationships would already have been shown to be different. However, since the intercept value is related to the coefficient of condition, it should be noted that all of the intercept values for the central Pacific fish were smaller than those for the eastern tropical Pacific fish by a factor of 10. These differences may not be real because the samples for the central Pacific contained larger individuals than did the samples for the eastern tropical Pacific.

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