

IMPROVED BOUNDS FOR ROW AGGREGATED LINEAR PROGRAMS:

A NOTE

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DRAFT FOR COMMENT

For any partition $\rho' = \{R'_\ell : \ell = 1, \dots, L'\}$, let $\{e_1, \dots, e_m\}$ be known positive numbers, and $\{q_1, \dots, q_{L'}\}$ known nonnegative numbers, such that u^* , the optimal dual variables in (1) satisfy:

$$\sum_{i \in R'_\ell} e_i u_i^* \leq q_\ell, \quad \ell = 1, \dots, L'. \quad (3)$$

Then Zipkin [3] shows:

$$\hat{z} - \xi_\alpha^- \leq z^*$$

where

$$\xi_\alpha^- = \sum_{\ell=1}^{L'} \left[\max_{i \in R'_\ell} \{(A_i \hat{x} - b_i)/e_i\} \right]^+ q_\ell$$

where \hat{x} is an optimal solution to (2).

To extend theorem 2 in [1] and theorem 2.1 in [2], consider the dual to (1):

$$\begin{aligned} z^* = \text{minimize } & ub \\ \text{s.t. } & uA \geq c \\ & u \geq 0 \end{aligned} \quad (4)$$

transformed to:

$$\begin{aligned} \text{maximize } & u(-b) \\ \text{s.t. } & u(-A) \leq -c \\ & u \geq 0 \end{aligned} \quad (5)$$

The partition ρ and ρ' , and the bounds (3) now apply to a column aggregated LP. The equivalent aggregate problem is:

$$\begin{aligned}
& \text{maximize } U(-\tilde{b}) \\
& \text{s.t. } U(-\hat{A}) \leq -c \\
& \quad U \geq 0
\end{aligned} \tag{6}$$

where U is an L -vector of variables. From proposition 2 in [3], and theorem 2.1 in [2], there exists a function $z(\theta)$, such that at θ^* ,

$$\bar{u}(-b) \leq z(\theta^*) \leq \hat{U}(-\tilde{b}) + \xi_{\alpha}^- \tag{7}$$

This is equivalent to:

$$\hat{z} - \xi_{\alpha}^- \leq -z(\theta^*) \leq \bar{ub} = z^* \tag{8}$$

Equation (8) states that Kallio's method, as used to tighten the upper bounds on column aggregate problems, can also be used to tighten the lower bounds for a row aggregated problem. The reader is referred to [1] and theorem 2.1 in [2] for details of how to calculate $-z(\theta^*)$.

LITERATURE CITED

- [1] M. Kallio. "Computing bounds for the optimal value in linear programming." *Naval Research and Logistic Quarterly* 24(2): 301-308. (1977)
- [2] R. Mendelssohn. "Improved bounds for aggregated linear programs." *SWFC Admin. Rep. 16H*, 1978, 19 p. (1978)
- [3] P. Zipkin. "Aggregation in linear programming." Ph.D. Dissertation, Yale University, New Haven, 189 p. (1977)

In [2], a result of Kallio [1] is extended in order to tighten Zipkin's [3] upper bound for column aggregated linear programs (LPs). In this note, it is shown that an identical procedure can be used to tighten Zipkin's lower bound for row aggregated LP's. The notation follows [3] throughout, and familiarity with the results and methods of [1, 2, 3] is assumed.

The primal LP to be solved is:

$$\begin{aligned}
 z^* &= \text{maximize} && cx \\
 &\text{subject to} && Ax \leq b \\
 &&& x \geq 0
 \end{aligned} \tag{1}$$

where $c = (c_j)$ is an n -vector, $b = (b_i)$ is an m -vector, $A = (a_{ij})$ is an $m \times n$ matrix, and $x = (x_j)$ is an n -vector of variables.

Let $\rho = \{R_\ell : \ell = 1, \dots, L\}$ be a partition of $\{1, \dots, m\}$, where $|R_\ell| = m_\ell$. Let f be a nonnegative m -vector whose components sum to unity. Define:

$$\begin{aligned}
 \tilde{b} &= (\tilde{b}_\ell); \quad \tilde{b}_\ell = f^\ell b^\ell \\
 \hat{A} &= (\hat{a}_{kj}); \quad \hat{a}_{kl} = f^\ell A_{kl}
 \end{aligned}$$

so that the row aggregated LP becomes:

$$\begin{aligned}
 z &= \max cx \\
 &\text{subject to} && \hat{A}x \leq \tilde{b} \\
 &&& x \geq 0
 \end{aligned} \tag{2}$$



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Enclosed is a short companion to the earlier paper I sent you, "Improved bounds for aggregated linear programs." The purpose of the note was to convince myself that the method to improve upper bounds for column aggregated LP's does work for row aggregated LP's. Again, comments, suggestions, etc., are most welcome.

With regards,

A handwritten signature in cursive script, appearing to read "Roy".

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