Southwest Fisheries Center Administrative Report H-79-3

COMPUTING STOCK SIZE INDICES FROM CATCH-PER-UNIT-EFFORT STATISTICS

Jerry A. Wetherall
Southwest Fisheries Center
National Marine Fisheries Service, NOAA
Honolulu, Hawaii 96812

February 1979

Purpose

This note summarizes some of the fundamental relationships and equations for computing annual stock size indices using catch-per-unit-effort (CPUE) statistics, and gives simple expressions for estimating the sampling variance of such indices. It is motivated by the general need for some measure of precision to accompany the usual point estimates of relative population size, and by the specific desirability of using statistical weights when analyzing the stock indices by least squares methods.

Following the definition of terms and the statement of basic relations and estimators, two application situations are discussed. In Case 1, the spatial and temporal coverage of the fishery in the stock region is complete each year; in Case 2, coverage is not complete each year because of expansion or contraction of the fleet's range of operations or because of the seasonal absence of fishing activity in certain zones.

Notation

N_i = Average stock size in year i in the defined stock region

N_{ijk} = Average stock size in year i in the j^{th} fishing zone and k^{th} fishing season

n_i = Number of potential fishing zones in the stock region in year i

r_i = Number of potential fishing seasons during the i^{th} year

ℓ_i = Actual number of seasons in which fishing took place in year i

m_{ik} = Actual number of fishing zones in which effort was exerted during the k^{th} fishing season of year i

p_{ijk} = Ratio of average stock size in the j^{th} fishing zone during the k^{th} season of year i to the average overall stock size during year i

$\hat{N}_{i(jk)}$ = Estimate of N_i based on observations from the j^{th} fishing zone during the k^{th} fishing season of year i

$\hat{u}_{i(jk)}$ = Stock size index corresponding to $\hat{N}_{i(jk)}$

$\hat{u}_i$ = Overall index of stock size in year i

$\hat{\sigma}^2_{\hat{u}_i}$ = Estimate of the variance of the overall index of stock size in year i

$\hat{\sigma}^2_{\hat{u}_{i(jk)}}$ = Estimated variance among the individual indices of stock size in year i

A_{ijk} = Area of the j^{th} fishing zone in which fishing occurred during the k^{th} season of year i

C_{ijk} = Catch taken in the j^{th} fishing zone during the k^{th} season of year i

f_{ijk} = Nominal fishing effort exerted in the j^{th} fishing zone during the k^{th} season of year i [adjusted for differences in fishing power, if any]

q = Catchability coefficient (assumed constant)

F_{ijk} = Instantaneous fishing mortality rate in fishing zone j during the k^{th} season of year i

D_{ijk} = Average density of fish in the j^{th} fishing zone during the k^{th} fishing season of year i

$CV(\hat{u}_i)$ = Coefficient of variation of the overall stock size index in year i

f_i' = Effective fishing intensity during year i

I_i = Concentration coefficient during year i

a_{jk} = Number of "standard" years when fishing effort was exerted in the "j^{th}" fishing zone during the "k^{th}" fishing season

$\hat{p}_{ijk}$ = Estimate of p_{ijk} based on statistics from the j^{th} fishing zone during the k^{th} season of year i

$\hat{p}_{jk}$ = Estimate of p_{ijk} based on an average of annual estimates derived from "standard" years when fishing covered the entire stock range and all fishing seasons

Basic Relations

We assume that the entire stock in question [or a constant fraction of some larger stock] occupies a defined stock region consisting in year i of n_i potential fishing zones and that there are r_i potential fishing seasons of equal duration. In the course of a particular year, fishing actually takes place during ℓ_i seasons ($\ell_i \leq r_i$), with m_{ik} ($m_{ik} \leq n_i$) fishing zones being occupied in the k^{th} season.

Following the standard deterministic theory of fishing, the catch in a particular season-zone stratum and year is described as the product of fishing mortality rate and stock density in the given stratum, i.e.,

$$C_{ijk} = F_{ijk}\, D_{ijk} = q\, f_{ijk}\left(\frac{N_{ijk}}{A_{ijk}}\right) \qquad \ldots (1)$$

We define

$$p_{ijk} = \frac{N_{ijk}}{\sum_{k=1}^{r_i}\left(\sum_{j=1}^{n_i} N_{ijk}\right)\Big/ r_i}$$

$$= \frac{N_{ijk}}{N_i}$$

so that $N_{ijk} = p_{ijk}\, N_i$ ... (2)

Substituting (2) in (1) we obtain

$$C_{ijk} = \frac{q\, f_{ijk}\, p_{ijk}\, N_i}{A_{ijk}} \qquad \ldots (3)$$

Now regarding catch as an observed random variable, there is an estimate of the absolute stock size, N_i, arising from each time period and geographical stratum actually fished in year i, viz.,

$$\hat{N}_{i(jk)} = \left(\frac{1}{q}\right)\left(\frac{C_{ijk}\, A_{ijk}}{f_{ijk}\, p_{ijk}}\right)$$

We drop the constant catchability coefficient, q, to obtain a corresponding index of stock size based on observations in the $(jk)^{th}$ stratum,

$$\hat{u}_{i(jk)} = \frac{C_{ijk}\, A_{ijk}}{f_{ijk}\, p_{ijk}} \quad .$$

The overall annual index of stock size is then computed as the average of the individual stratum estimates,

$$\hat{u}_i = \sum_{k=1}^{\ell_i} \sum_{j=1}^{m_{ik}} \hat{u}_{i(jk)} \Bigg/ \sum_{k=1}^{\ell_i} m_{ik}$$

$$= \sum_{k=1}^{\ell_i} \sum_{j=1}^{m_{ik}} \frac{C_{ijk} A_{ijk}}{f_{ijk} p_{ijk}} \Bigg/ \sum_{k=1}^{\ell_i} m_{ik} \qquad \ldots (4)$$

Note that all elements of (4) are observed except for p_{ijk}, which must be estimated (see below).

The sampling variance of the stock size index is estimated by

$$\hat{\sigma}^2_{\hat{u}_i} = \hat{\sigma}^2_{\hat{u}_{i(jk)}} \Bigg/ \sum_{k=1}^{\ell_i} m_{ik} \qquad \ldots (5)$$

where

$$\hat{\sigma}^2_{\hat{u}_{i(jk)}} = \sum_{k=1}^{\ell_i} \sum_{j=1}^{m_{ik}} \left(\hat{u}_{i(jk)} - \hat{u}_i\right)^2 \Bigg/ \left(\sum_{k=1}^{\ell_i} m_{ik} - 1\right)$$

and the coefficient of variation of $\hat{u}_i$ is

$$CV(\hat{u}_i) = \hat{\sigma}_{\hat{u}_i} \Big/ \hat{u}_i \quad .$$

An alternate way to compute the stock size index is as

$$\hat{u}_i = \frac{\sum_{k=1}^{\ell_i} \sum_{j=1}^{m_{ik}} C_{ijk}}{\sum_{k=1}^{\ell_i} \sum_{j=1}^{m_{ik}} \frac{f_{ijk} p_{ijk}}{A_{ijk}}}$$

$$= \frac{C_{i\cdot\cdot}}{f_i'}$$

$$= \frac{C_{i\cdot\cdot}}{I_i \sum_{k=1}^{\ell_i} \sum_{j=1}^{m_{ik}} \frac{f_{ijk}}{A_{ijk}} / n_i}$$

where f_i' is the effective fishing intensity in the i^{th} year and I_i is the corresponding concentration coefficient.

Note that I_i is here defined as

$$I_i = \frac{n_i \sum_{k=1}^{\ell_i} \sum_{j=1}^{m_{ik}} \frac{f_{ijk}}{A_{ijk}} p_{ijk}}{\sum_{k=1}^{\ell_i} \sum_{j=1}^{m_{ik}} \frac{f_{ijk}}{A_{ijk}}} .$$

Estimating p_{ijk}

The index described by (4) and its variance estimate given by (5) apply to all situations regardless of how completely the fishing fleet covers the stock region. However, the calculation of the p_{ijk} will differ depending on whether coverage is complete or not.

Case 1: Complete Coverage

When $\ell_i = r_i$ and $m_{ik} = n_i$ for all i and k, then it is possible to estimate the p_{ijk}'s individually as

$$\hat{p}_{ijk} = \frac{\left(\frac{C_{ijk}A_{ijk}}{f_{ijk}}\right)}{\sum_{k=1}^{\ell_i}\left(\sum_{j=1}^{m_{ik}}\frac{C_{ijk}A_{ijk}}{f_{ijk}}\right)\Big/\ell_i} \quad \ldots (8)$$

$$\left(\frac{n_i}{m_{ik}}\right)$$

and in this event the stock size index reduces to

$$\hat{u}_i = \sum_{k=1}^{\ell_i}\left(\sum_{j=1}^{m_{ik}}\frac{C_{ijk}A_{ijk}}{f_{ijk}}\right)\Big/\ell_i$$

$$\left(\frac{n_i}{m_{ik}}\right)$$

Case 2: Incomplete Coverage

When $\ell_i < r_i$ and/or $m_{ik} < n_i$ for some i, then it is not possible to estimate the p_{ijk}'s for those years. In this situation Honma (1974) invokes the assumption that the temporal and spatial distribution of the stock is constant each year, i.e., $p_{ijk} = p_{jk}$ for all i, and suggests estimating the p_{jk}'s using data from "standard" years when coverage was relatively complete. Supposing there are a_{jk} "standard" years when the $(jk)^{th}$ stratum was fished, we estimate the average p_{jk} for that stratum by

$$\hat{p}_{jk} = \sum_{i=1}^{a_{jk}}\hat{p}_{ijk}\Big/a_{jk}$$

where $\hat{p}_{ijk}$ is given by (8).

In practice, where there are a sufficient number of "standard" years we usually require that a_{jk} exceed some specified minimum value (e.g., $a_{jk} > 2$) to reduce the effects of extreme observations. If a_{jk} does not satisfy this condition we ignore observations from that stratum. [It is clear that the "j^{th}" fishing zone in one year may not be the same as the "j^{th}" fishing zone in another year; similarly for fishing seasons. But it is understood that in computing a particular $\hat{p}_{jk}$ we must use statistics from the same zone and season.]

LITERATURE CITED

Honma, Misao. 1974. Estimation of overall effective fishing intensity of tuna longline fishery. Bull. Far Seas Fish. Res. Lab. 10:63-85.