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Discount Factors and Risk Aversion in Managing Random Fish Populations

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Introduction

The standard literature (see Clark 1976, 1980; Anderson 1977; and references therein) on dynamic, deterministic fishery optimization models has stressed the importance of the discount factor (1/1+i, where i is the interest rate) in determining optimal harvesting strategies. The discount factor has been emphasized because an optimal harvesting strategy depends continuously on the discount factor, and a stock will be optimally depleted if the rate of growth of the population is less than the rate of discount (Reed 1974; Clark 1976). Related results have been established for stochastic harvesting models (Reed 1974; Mendelssohn 1980b; Mendelssohn and Sobel 1980), though in stochastic models a stock may become depleted even if it isn't harvested to depletion.

The presumed sensitivity of an optimal harvesting strategy to changes in the discount factor is important since a standard procedure for representing increased risk to a manager of a fishery in face of uncertainty is to decrease the discount factor. Even at first glance this is a naive approach, since it assumes a constant attitude towards risk regardless of the initial population size or harvest decision.

What would appear to be a more sophisticated approach towards risk is to assess the manager's utility function and use this as the objective function. This approach to managing fisheries and other natural resources is given in Hilborn and Walters (1977), Keeney (1977), and Holling (1978); a general discussion can be found in Keeney and Raiffa (1976).

In this paper, several of these strands are examined for one empirical stochastic model of a fishery. In particular, the actual sensitivity of an optimal harvesting strategy to changes in the discount factor are compared to its sensitivity to changes in the degree of risk aversion in the utility function. Further changes in the population dynamics when following an optimal harvesting policy are examined as both the discount factor and the degree of risk aversion vary. The results suggest that (1) policy often is insensitive to changes in the discount factor over a range of values most likely to be found in practice; and (2) that, while a discount factor adequately represents intertemporal preferences for a certain dollar now versus a certain (expected) dollar in the future, it does not adequately capture the intertemporal preference for the gamble involved in obtaining an expected dollar in the future. Moreover, as will be shown, since risk-averse utility functions can be viewed as total revenue curves with marginal price sensitive to supply, the results suggest that highly supply-sensitive prices will exert a stabilizing effect on the optimal harvest of fish, when the fishery is assumed to be either economically or biologically stochastic. Since most if not all fisheries are stochastic, the usual assumption of perfect competition in fishery economic models and the lack of attention to risk preferences limit the usefulness of the results derived from these other models.

External decisions for a fishery, such as entry decisions and enhancement projects, depend on the valuation of the fishery, that is the discount factor used, the utility function chosen, and the sample-paths of the random harvest stream. The fishery manager usually can control only the last aspect. The results of this paper suggest that under optimal management, external decisions will not be strongly affected by this choice of policy.

Finally, as the degree of risk-aversion increases, the amount harvested is not strictly non-decreasing for all population sizes. This is seemingly in contradiction with the results of Cropper (1976); however, an intuitively appealing explanation for this result is given.

The Model

The model to be analyzed is a stochastic version of a Ricker spawner-recruit curve put forward by Mathews (1967) for salmon runs in Bristol Bay. Let x_t be the recruits in period t, and y_t the spawners. If x is observed and a decision y is taken, a one-period utility g(x, y) is received. Without loss of generality, g(x, y) can be the expected value (over the random

theta variable θ) of some other utility $h(x, y, \theta)$. Hence the model indirectly includes economic uncertainty.

beta The utility in period t is discounted by a factor β, $0 \leq \beta < 1$. The recruits in period t+1 are a random function of the spawners in period t and a random variable d,

$$(1) \quad x_{t+1} = d\ 4.084\ y_t \exp\{-0.8\ y_t\};\ \ln d \sim N(0,\ \ )$$

The problem is to maximize the expected discounted utility:

$$(2) \quad \text{maximize} \sum_{t=1}^{\infty} \beta^{t-1} g(x_t, y_t)$$

$$\text{s.t.} \quad x_t \geq y_t \geq 0;\ (1).$$

Computational techniques for solving (2) and related policy questions are discussed in Mendelssohn (1980b). A 100-point grid is used with no absorbing (zero) population size.

Two different utility functions are examined for g(x, y). The first is

lambda $v_1 = 0.5 \ln(x_t - y_t)$ while the second is $v_2 = (x_t - y_t)^{\lambda}$, for λ = 0.05, 0.1,

Fig. 1 0.25, 0.5, 0.75, 0.9, 0.95, 1.0. These utility functions are plotted in Fig. 1.

Utility functions are said to be absolute risk averse (Merton 1969, 1971; Keeney and Raiffa 1976) if $-u''(z)/u'(z) > 0$ for a function $u(z)$. The Pratt measure of relative risk aversion is $-u''(z)z/u'(z)$. Basically, a utility function is risk averse if a certain return is preferred to a lottery with equal or greater value in expectation.

The two utility functions v_1 and v_2 are members of the HARA (hyperbolic absolute risk averse) family of utility functions (Merton 1969, 1971). In particular, if R_1 is the measure of absolute risk aversion, and R_2 is the measure of relative risk aversion, then for v_1, $R_1 = 0.5/z$ and $R_2 = 1$; for v_2, $R_1 = (1- \;)/z$ and $R_2 = 1- \;$, where $z = x-y$ is the amount harvested. Both v_1 and v_2 have constant relative risk with respect to the amount harvested. However, it is clear that the relative risk aversion for v_2 changes linearly with λ. Hence λ is a measure of the relative degree of risk aversion. For λ approaching zero, the manager is totally risk averse, while for $\lambda = 1$ the manager is risk neutral. The utility function v_1 can be seen to be the limiting case of v_2 as λ approaches zero.

The utility functions v_1 and v_2 are not as rich as they might be given that they only depend on (x_t, y_t) through $z_t = x_t - y_t$. However, there exists little empirical basis for choosing more complicated utility functions that can be given meaningful, practical interpretations. Both v_1 and v_2 can be viewed as total revenue curves. Taking derivatives $v_1' = 0.5/z$ ($v_2' = \lambda z^{\lambda-1}$) and $v_1'' = -0.5/z^2$ (resp. $\lambda(1-\lambda)z^{\lambda-2}$). The first derivative is the marginal price for a supply of z units of fish, with the marginal price decreasing with increasing supply. Therefore, using v_1 and v_2 as the one-period utility functions can be viewed as examining the effect of dropping the usual economic assumption of perfect competition. This is treated analytically in some in detail in Mendelssohn and Sobel (1980).

Results

Optimal harvesting policies and optimal population dynamics were calculated for both classes of utility functions for discount factors of 0.99, 0.95, 0.925, 0.9, 0.85, 0.80, and 0.70. These discount factors are equivalent to interest rates of 1, 5.23, 8.1, 11.1, 17.65, 25, and 42.86%. Results are shown only for the utility function $v_1 = 0.5 \ln (x-y)$ and several of the extreme values of the discount factor. The results using v_2 with any value of λ are nearly identical as to those presented for v_1, and the graphs change uniformly

Fig. 2 with β, so the graphs shown represent the extreme changes. Figs. 2(a)-(d) present the results.

Fig. 2(a) shows an optimal harvesting strategy, and Fig. 2(b) shows the optimal number of spawners for $\beta = 0.99$, 0.925, and 0.70. Besides the fact that visually the curves are extremely similar, between $\beta = 0.99$ and $\beta = 0.70$ the graphs are identical at 50 out of 100 states and differ by only one-grid point at the rest of the states. Moreover, it can be shown that this result remains valid for much finer grids, hence the two policies can be made to be extremely close together.

Fig. 2(c) shows the long-run (ergodic) population sizes as a rough measure of the effect each of the optimal policies has on the population dynamics. There is a slight shift to smaller population sizes as β decreases. However, the cumulative ergodic harvest distribution for the three optimal policies are virtually indistinguishable.

Fig. 2(d) shows an optimal value function, $\sum_{t=1}^{t} \beta^{t-1} v_1(t)$ when starting in each state, for $\beta = 0.99$ and $\beta = 0.925$. Despite the fact that almost identical sample paths are generated by the two optimal harvesting strategies, the change in the valuation of the sample paths is dramatic.

There has been much discussion in the fisheries literature on the appropriate value of the discount factor (Walters 1975; Clark 1976; Walters and Hilborn 1976; Holling 1978). This discussion can be divided into two components--the discount factor that should be used by an external agency such as a government agency (say to compare the benefits derived from a stream enhancement project) or a private individual (when making an entry decision, for example) to evaluate the fishery, and that used by the fishery manager to set policy, bearing in mind the effects harvest policy will have on the former decisions. If the previous empirical results are robust, they suggest that the fishery manager need not worry greatly about the exact choice of the discount factor used for determining policy, and that this choice will have little bearing on entry decisions or enhancement projects. This is because a nearly optimal expected present value will be found over a broad range of discount factors that might be used by the external agents.

Fig. 3 Figs. 3(a)-(e) show similar results for v_2 with $\beta = 0.95$ and $\lambda = 1.00$ and $\lambda = 0.05$. Runs were performed for $\lambda = 1$, 0.95, 0.9, 0.8, 0.5, 0.25, and 0.05 and for different values of β. The results were not sensitive to the particular value of β used. The results changes uniformly with λ, hence the curves for $\lambda = 1.00$ and $\lambda = 0.05$ represent the extreme curves found.

Fig. 3(a) shows an optimal harvesting policy for each of the two values of λ. For $\lambda = 0.05$, more is harvested at lower population sizes, while less is harvested at larger population sizes. The crossover point is at a population size of 1.89×10^6, which is where the equation $\bar{d}$ 4.082 y exp {-0.8 y} has a value equal to $y = 1.89 \times 10^6$. Thus in expectation, an individual just replaces itself. Fig. 3(b) shows an optimal number of spawners. Fig. 3(c) shows the ergodic population densities when following the two optimal policies, and Fig. 3(d) shows the cumulative ergodic harvest distributions when following the two optimal policies. The cumulative harvest distributions differ by as much as 8%, and at the most likely harvest sizes, differ by about 5-6%. These differences are small but significant--the lower values of λ can be seen to produce a "smoother" harvest in the long run.

These results can best be explained by considering the economic interpretation of the utility functions. At $\lambda = 1$, the price does not vary with supply (in fact it is $p = 1$, which can be viewed as the normalized price), therefore a harvest policy is based entirely on being as close as possible to the point of largest overall expected growth. However, at $\lambda = 0.05$, at low populations, the higher price (resp. the higher risk) makes it desirable to harvest some now (resp. the risk makes it desirable to obtain some amount of certain return now). At higher population levels, a balance must be maintained between a desirable price and future growth. For $y \geq 1.89$, the expected value of x_{t+1} is less than y_t. Harvesting more now lowers the present marginal price per unit of fish, but increases the probability (resp. decreases the risk) of larger population sizes next period (resp. of small, unprofitable harvests next period).

The problem of unsmooth harvests from random populations is discussed in Mendelssohn (1976, 1980a, 1980b), Beddington and May (1977), and May et al. (1978). These empirical results suggest that unsmooth harvests have been found in the past because no effort has been made to include either attitudes towards risk, or marginal prices that are sensitive to supply. A reasonable conjecture is that all other things being equal, for a properly managed random fishery, wider fluctuations in catch will be expected either if outside supplies affect prices or if the entire quota cannot be taken by the fleet. Conversely, smaller fluctuations will be expected when the marginal price is very sensitive to the supply of the managed stock of fish only. Inventories can be viewed as an outside supply that affects price, and they should have a very destabilizing effect if not taken into account in setting management policy for biologically random fisheries. This is precisely what has been seen in the salmon fishery in Alaska.

The solution of problem (2) offers some insights into relative risk aversion and planning horizons when optimal policies are followed. This is because (2) can be solved by solving the following sequence of recursive equations:

$$f_n(x) = \max_{0<y<x} \{g(x, y) + \beta E f_{n-1} (s[y, d])\}$$

where n denotes the number of periods remaining in the planning horizon. For problem (2), each $f_n(\cdot)$ is concave and continuous (Mendelssohn and Sobel 1980), and hence each is a risk averse utility. Each $f_n(x)$ is the utility of having x units of fish when there are n periods left when following <u>an optimal policy</u>. Hence $f_n'(x)$ is the marginal value of one additional unit of fish with n periods remaining, and the two risk averse measures R_1 and R_2 can be calculated as n changes (derivatives are approximated by finite differences). The measure R_2 reflects the relative risk aversion faced by the decisionmaker given the actual sample paths that will occur when following an optimal policy.

Table 1 Table 1 gives the values of R_2 for several values of λ and for n = 2, 3, ..., 10. What is noticeable is that the relative risk for f_n is much less than that for v_2, that it increases with n, and that it quickly becomes stationary after 3-5 periods. An optimal harvesting policy reduces the relative risk of the decisionmaker. Moreover, the effective planning horizon for these problems appears to be 3-5 years, after which both policy, risk and preferences remain stationary.

The measure of relative risk R_2 can be seen to be almost a measure of elasticity of the present marginal value of a unit of fish. As an optimal policy "smooths out" the supply of fish, the elasticity of the value per unit of fish decreases.

In conclusions, these results suggest that properly assessing attitudes towards risk is more important than choosing an appropriate discount factor when managing a random fishery. Deterministic models that ignore fluctuations in catch, and the effects these fluctuations have on income, can lead to mismanagement of fisheries.

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Table 1. Changes in relative risk aversion with changes in planning horizon length when following an optimal policy.

	$\lambda = 1$	$\lambda = 0.5$	$\lambda = 0.05$
n = 2	0.1430	0.0518	0.045
n = 3	0.1684	0.303	0.423
n = 4	0.1684	0.325	0.423
n = 5	0.1684	0.331	0.423
n = 6	0.1684	0.331	0.423
n = 7	0.1684	0.331	0.423
n = 8	0.1684	0.331	0.423
n = 9	0.1684	0.331	0.423
n = 10	0.1684	0.331	0.423

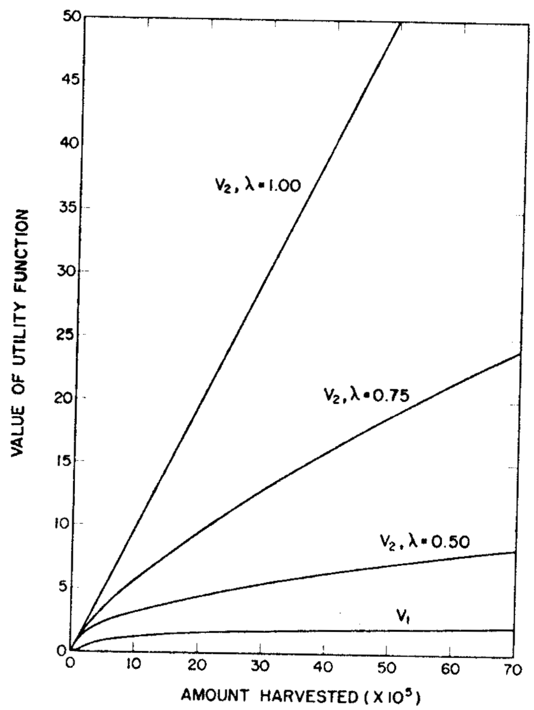

Fig. 1. Graph of the different utilities function.

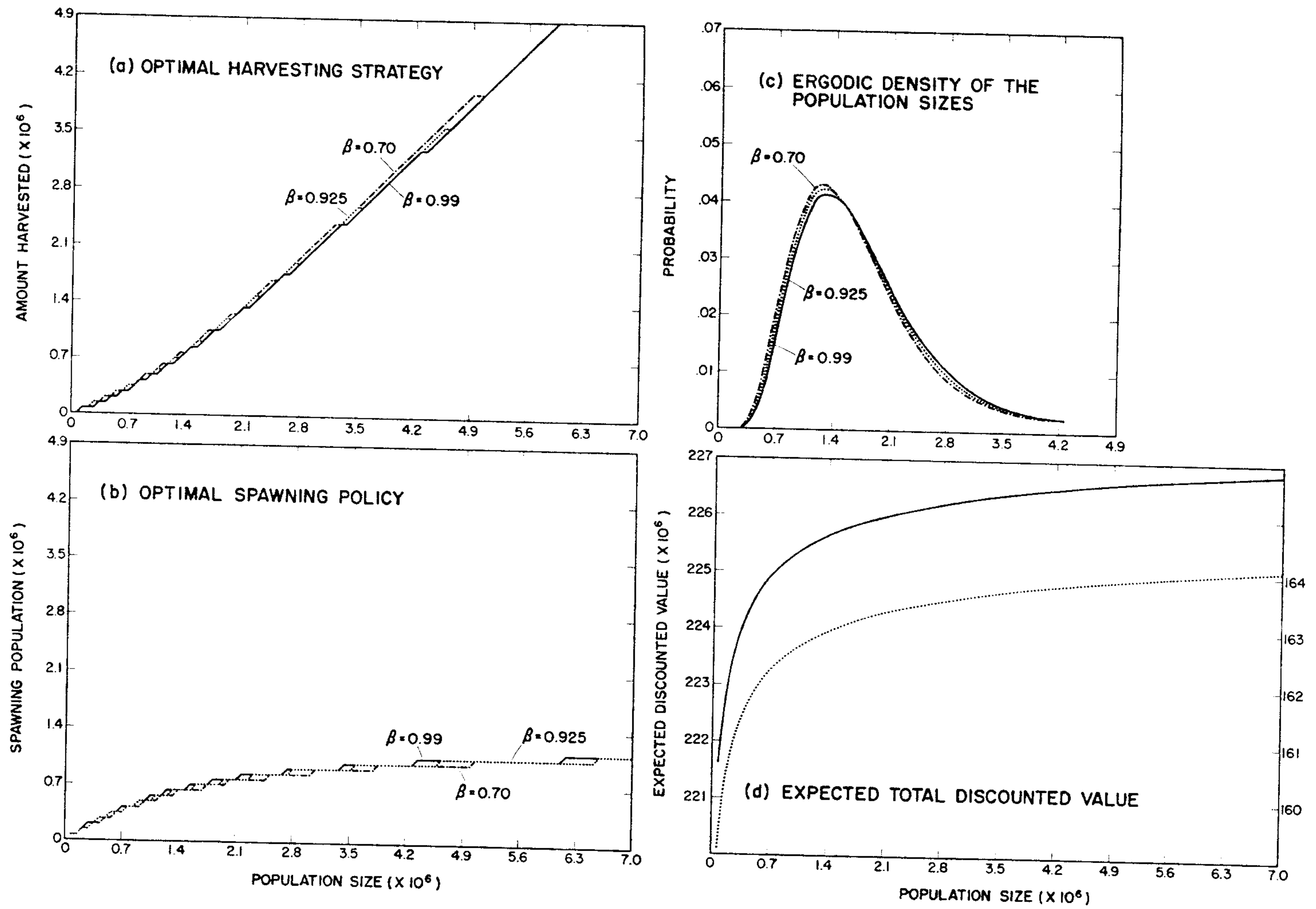

Fig. 2. Effects on an optimal policy and optimal value from varying the discount factor.

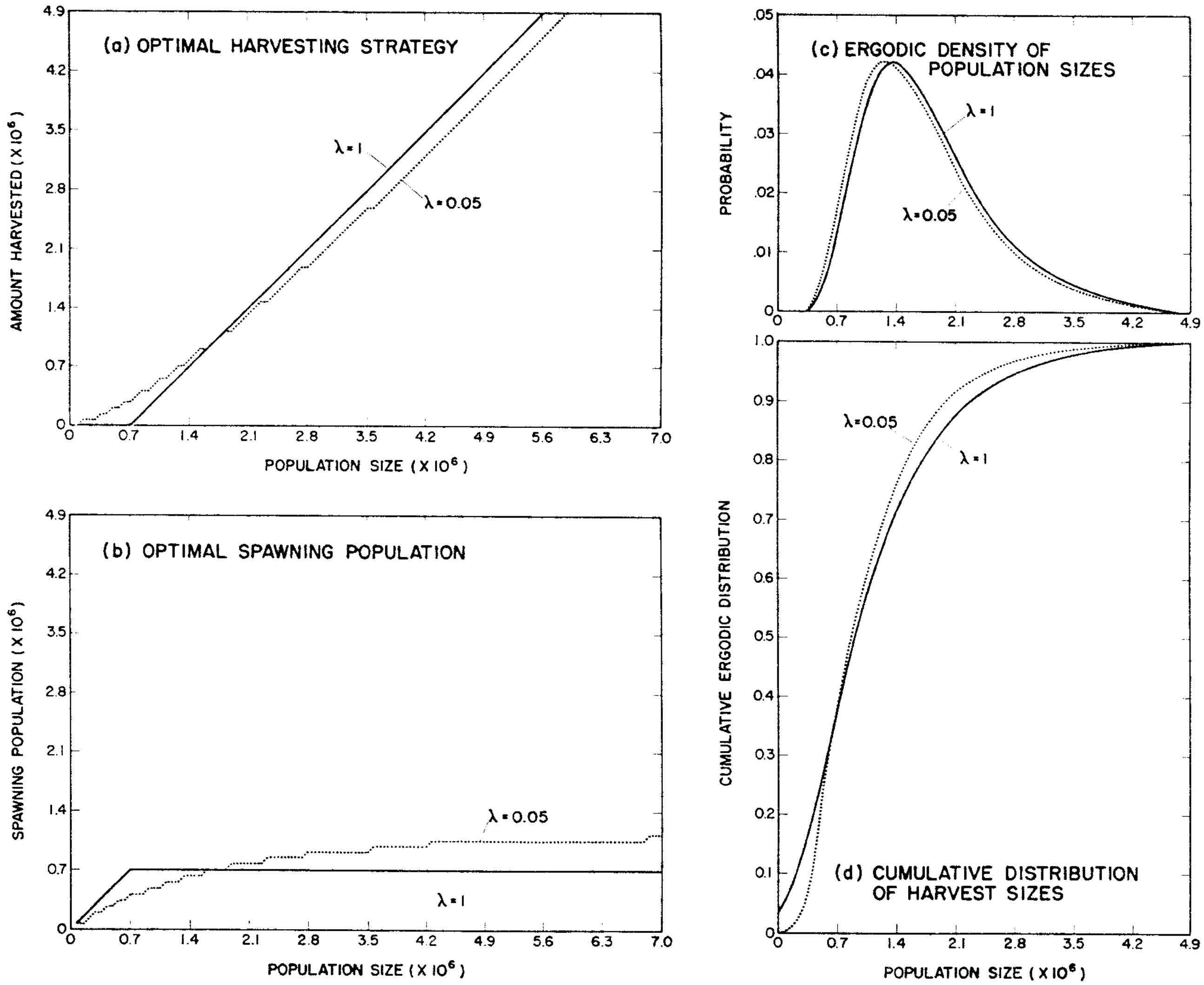

Fig. 3. Effects on an optimal policy and optimal value from varying the degree of risk aversion.